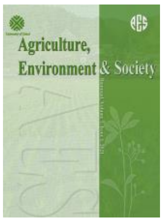
Agriculture, Environment & Society (2022) Volume 2, Issue 1, P. 103-110

Agriculture, Environment & Society

Journal homepage: www.aes.uoz.ac.ir



A fuzzy data envelopment analysis for assessing sustainability in potato farms of Kabodarahang village

Zahra Ghaffari Moghadam [*a], Seyed Mehdi Hosseiny [b], Somayyeh Mirshekari [a]

[a] Research Institute of Zabol, Zabol, Iran
[b] Department of Agricultural Economics, Management and Economic Faculty, University of Sistan and Baluchestan, Zahedan, Iran

ARTICLE INFO

Article history:
Received: 4 April 2022
Accepted: 10 May 2022
Available online: 1 June 2022

Keywords:
α- cut technique
Decision making units
Efficiency
Fuzzy context
Fuzzy DEA models

ABSTRACT

The disparity in agricultural production unit performance is a critical and fundamental issue that necessitates the implementation of required programs and policies to ensure an equilibrium in the distribution of resources and seeds in order to increase efficiency. Enhancing efficiency can result in economic growth and development in the agricultural sector, as well as rural development. In recent years, the potato has been regarded as the dominant crop in Kabodarahang; policy formulation based on production efficiency has become a necessity, given the importance of production and the rational use of seeds in potato production. Thus, the efficiency, effectiveness, and productivity of production units were determined under uncertain conditions using various level-cut and fuzzy data envelopment analysis methods for potato farms in Kabodarahang; the most efficient units were selected. The result indicates that 14% of producers are efficient or near-efficient. Additionally, 75% of producers operate at a level of efficiency between 70% and 100%. In total, unit 6 is the most productive and stable of the other units. As a result, this unit is chosen as the best producer. Efficiency analysis at various levels reveals that producers operate at a high level of efficiency. Therefore, to increase production, manufacturing technology should be enhanced. Thus, politicians and policymakers should take into account new technologies for planting, growing, and harvesting.

Highlights

- As a result of the disparity in agricultural production unit performance, programs and policies must be implemented to ensure an equilibrium in the distribution of resources and seeds.
- Increasing efficiency can lead to increased agricultural and rural economic growth.
- Given the importance of production and the rational use of seeds in potato production, policy formulation based on production efficiency has become necessary.
- 14% of potato farms in KabodarAhang are efficient or near-efficient, 75% of producers are between 70% and 100% efficient.
- Politicians and policymakers should consider new planting, growing, and harvesting technologies.

1. Introduction

Data Envelopment Analysis (DEA) can be considered a robust performance management tool to evaluate decision-making units (DMUs) capable of benchmarking the unit's performance against other competitors and deciding on a better future based on its outcomes. This tool measures the relative efficiency of DMUs with the same inputs and outputs and, accordingly, specifies efficient and inefficient units of performance (Alinezhad et al., 2018). In low-income countries, agricultural production, indirectly related to people's lives, is recognized as a strategic product. Several studies indicate the specific status of this product in the countries mentioned above (Mardani and Ziaee, 2016). Potato products are of particular importance because of their high nutritional value. Production of this

---

[*] Corresponding author
Email: zahraghafari@uoz.ac.ir

http://dx.doi.org/10.22034/aes.2022.336019.1029

product in developed countries is greater than in developing countries (Sepehrdoust and Emami, 2017).

Potato production in Iran is about 5 million tons, and its per capita consumption is about 45 kg (Ministry of Jahad Agriculture, 2011). With 23.5% of the country's potato production, Hamedan province has first place in the production of this crop. Besides, in the 2012-2013 crop year, 16.63% of the total potato croplands were allocated to Hamedan. This province ranked first with 21.72% of the Sepehrdoust and Emami, 2017). The Kabodarahang region in this province, whose farmers pay a lot of attention to the potato crops, is the leading producer of potatoes in the country and the exporter of this product to neighboring countries (Sepehrdoust and Emami, 2017). In this regard, rational planning to manage the production of this product and the way to use the production resources to increase production efficiency is inevitable.

DEA is a tool to measure the relative efficiency of units under evaluation that have multiple inputs and outputs. DEA was first described by Charnes et al. in 1978 and is widely used in Multi-Criteria Decision Making (MCDM). It considers a nonparametric linear programming technique to evaluate relative efficiency in decision-making units, using multiple inputs and outputs. Over the last three decades, various DEA models have been used to measure technical efficiency or effectiveness in different units. Most studies have compared the preference for technical efficiency with technical performance (Chiou et al., 2010). In many applications, they have encountered inaccurate or ambiguous data; our knowledge of the production process is inaccurate. This has led to combining the data envelopment analysis model with fuzzy set theory (Chiou et al., 2010). Bellman and Zadeh introduced the fuzzy concept to measure the efficiency based on inaccurate data in the decision-making process (Bellman and Zadeh, 1970).

Sengupta (1992) was the first to introduce a fuzzy programming method that was used in DEA models for multiple inputs. Cooper et al. (1999) proposed a model with the ability to consider inputs and outputs as either crisp or interval values. Guo and Tanaka (2001) considered data in the form of fuzzy Triangular and then, by applying the α-cut on constraint with comparison, determined the efficiency interval for each DMU. Hatami Marbini et al. (2017) proposed a novel fully fuzzy data envelopment analysis (FFDEA) approach, in which all the variables are considered fuzzy, including input and output data and efficiency scores. Moreover, a lexicographic multi-objective linear programming (MOLP) approach is suggested for solving the fuzzy models. Chiou et al. (2010) proposed two novel integrated data envelopment analysis (IDEA) approaches, comprising ICCR and IBCC, to jointly measure the technical efficiency and service effectiveness for bus transit services under constant and variable returns to scale technologies. It is demonstrated that the proposed novel IDEA approaches have higher benchmarking power than the conventional separate DEA. As Kumar et al. (2014) investigated, DEA provides a robust approach to supplier selection problems. Weber et al. (2000) proposed MOP and DEA to evaluate suppliers. Farzipoor Saen (2009) proposed a DEA model for supplier selection in the presence of both undesirable outputs and inaccurate data. Sengupta (1992) integrated fuzzy inputs and outputs in the DEA model by defining boundary levels in the objective function and constraining them. Hatami-Marbini et al. (2011) proposed a linear programming (LP) model with fuzzy parameters to measure the fuzzy efficiency of the DMUs. For different α levels, Esmaeili (2012) addressed an enhanced Russell measure (ERM) model with interval Data to evaluate the efficiency of the decision-making units. Azadi et al. (2015) used concept fuzzy in sustainable supply chain management to find the best supplier, and they created envelopment analysis using the Russell measure model. Qin and Liu (2010) developed a fuzzy random DEA (FRDEA) model, where randomness and fuzziness exist simultaneously. Many researchers have also proposed and applied various fuzzy DEA models (e.g., Nandy and Singh, 2021; Mardani et al., 2020; Dadmand and Naji Azimi, 2018; Hatami Marbini et al., 2012; Tlig and Hamed, 2017).

As mentioned above, several studies have examined performance using data envelopment analysis and fuzzy data envelopment analysis, and most of these studies have only obtained the concept of efficiency for decision units, but in this study, Using the proposed model, will obtain the efficiency, effectiveness, and productivity of the production units in conditions of uncertainty, using different levels of alpha for potato fields in Kaboudar Ahang, and the most efficient units will be selected.

Table 1. The nomenclatures.

Variable	Definition	Variable	Definition
j	The index of DMUs1,...,j n	ε_1 and ε_2	Predetermined acceptable levels of the possibility of objective function
r	The index of outputs1,...,r s	$\bar{\mathcal{G}}$	The maximum value denoting return function of effectiveness
i	The index of inputs1,...,i m	$\bar{\mathcal{F}}$	The maximum value denoting return function of efficiency
DMU_0	The DMU under evaluation	$\tau_1 \dots . \tau_5$	Predetermined acceptable levels of the possibility of constrains
y_{rj}	The rth output of jth DMU	$\mathrm{v_i}$	The weight for the ith input
x_{ij}	The ith input of jth DMU	$\propto . \beta . \mu_i . f_r$	The dual variables
$\mathrm{y_{ro}}$	The rth output of DMU_0	g_{to}	The tth goal of the DMU_0
x_{io}	The ith input of DMU_0	η_t	The weight of the tth goal
$\mathrm{u_r}$	The weight for the rth output		

2. Material and method

In most studies that measure performance, efficiency can only be achieved using a fuzzy concept. The proposed model measures effectiveness, efficiency, and productivity in a fuzzy context. DEA can be considered as a method to determine the efficiency of a set of decision-making units (DMUs) based on the measure of outputs rather than inputs.

Although the typical method of DEA can be considered a powerful tool for measuring the efficiency of decision-making units, it also has some limitations. One of the most

important of these limitations may be the high sensitivity of this method to the value change in input and output data or the uncertainty associated with these data, So that the ranking and stimulation of unit efficiency level can be changed entirely with the slightest change in the values of input and output data (Kao and Liu, 2003). Due to the sampling errors or the use of the central tendency indexes, the uncertainty in the data for estimation of the DEA model in the agricultural sector is inevitable, and the necessity of using models that are capable of controlling the changes resulting from unreliable data (Toma et al. 2015).
The indices, parameters and variables that will be used in this study are described in Table 1.

The model used in the current study was developed by Azadi et al. (2015). Assumptions are evaluated by the Decision-making unit (DMU). Any DMU would consume different inputs and produce different outputs. Consider $x_{ij}(i = 1.2. \ldots . m)$ and $y_{rj} = (1.2. \ldots . s)$ demonstrating data fuzzy input and output ofDMU_j; $\mathrm{u_{r.}}$ $\mathrm{v_i}$ denote the rth output weight and ith input weight, respectively. N is the number of DUMs, (j=1, 2… n). Assuming all the inputs and outputs are positive, Esmaeili (2012) proposed the dual of ERM as follows:

$$
\begin{aligned}
&\text{Max } E = \alpha - \beta && (1)\\
&\text{s.t } \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0. && j = 1. \ldots . n\\
&v_i x_{io} - \mu_i \le \frac{1}{m}. && i = 1 \ldots . m\\
&\frac{\propto}{s} - \mathrm{u_r y_{ro}} + \mathrm{f_r} \le 0. && \mathrm{R} = 1 \ldots . \mathrm{s}\\
&\sum_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{\mu_i} - \sum_{\mathrm{r=1}}^{\mathrm{s}} \mathrm{f_r} - \beta \le 0\\
&\propto . \beta . \mathrm{\mu_i} . \mathrm{f_r} . \mathrm{u_{r.}} \mathrm{v_i} \ge 0
\end{aligned}
$$

The dual variable ∝ in the first constraint of the primal model implies the average output efficiency. The dual variables β and $\mathrm{f_r}$ associated with Equation (1) have no practical implications. They have only been developed to transform a non-linear model into a linear one. The $\mathrm{E} = [0.1]$ denotes the efficiency score of $\mathrm{DUM_o}$. If the objective function of the Equation (1) equals 1 known as the$\mathrm{DUM_o}$, it is a relatively efficient unit. Otherwise, the DUM is a relatively inefficient unit. In many cases, measuring the effectiveness of each DUM can be as important as the efficiency measurement. Effectiveness refers to how much a company can meet its predetermined goals. The traditional DEA models fail to measure the effectiveness of DUMs. In this paper, we define the effectiveness of a DMU as the ratio of the output to the predetermined goals as follows (Azadi et al., 2015):

$$
\text{Effectiveness} = \frac{\text{output}}{\text{goal}} \qquad (2)
$$

To this end, a new model is proposed. To measure both the efficiency and effectiveness of the $\mathrm{DMU_o}$ the Equation (1) is converted as follows (Azadi et al. 2015):

$$
\begin{aligned}
&Max\ \alpha - \beta + \left(\frac{\sum_{\mathrm{r=1}}^{\mathrm{s}} \mathrm{u_r y_{ro}}}{\sum_{\mathrm{t=1}}^{\mathrm{T}} \eta_{\mathrm{t}} \mathrm{g_{to}}}\right) && (3)\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0. && j = 1 \ldots . n\\
&v_i x_{io} - \mu_i \le \frac{1}{m}. && i = 1 \ldots . m\\
&\frac{\propto}{\mathrm{s}} - \mathrm{u_r y_{ro}} + \mathrm{f_r} \le 0. && \mathrm{R} = 1 \ldots . \mathrm{s}\\
&\sum_{i=1}^{m} \mu_i - \sum_{r=1}^{s} f_r - \beta \le 0\\
&\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{t=1}^{T} \eta_t \mathrm{g}_{t\mathrm{j}}} \le 1. && \mathrm{j} = 1 \ldots . \mathrm{n}\\
&\propto . \beta . \mu_i . f_r . u_{r.} v_i . \eta_t \ge 0 && \forall \mathrm{i.r.t.}
\end{aligned}
$$

Where the *t* goal of the DMU_o is denoted as g_{to}. The η_t is weight of the *t* goal. Equation (3) can be rewritten as follows:

$$
\begin{aligned}
&\text{Max } P = \left[\textstyle\sum_{t=1}^{T} \eta_t \mathrm{g}_{to}(\alpha - \beta)\right] + \sum_{r=1}^{S} u_r y_{ro} && (4)\\
&\qquad \text{s.t}\\
&\sum_{t=1}^{T} \eta_t \mathrm{g}_{to} = 1\\
&\sum_{r=1}^{S} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0. && j = 1 \ldots m\\
&v_i x_{io} - \mu_i \le \frac{1}{m}. && i = 1 \ldots . m\\
&\frac{\propto}{S} - u_r y_{ro} + f_r \le 0. && \mathrm{R} = 1 \ldots . \mathrm{s}\\
&\sum_{i=1}^{m} \mu_i - \sum_{r=1}^{s} f_r - \beta \le 0\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{t=1}^{T} \eta_t g_{tj} \le 0. && j = 1 \ldots n\\
&\propto . \beta . \mu_i . f_r . u_{r.} v_i . \eta_t \ge 0 && \forall \mathrm{i.r.t.}
\end{aligned}
$$

The $P \in [0.2]$ shows the productivity score of DMU_o. If the optimal value of the Equation (4) equals 2, the $DMU_o$is relatively productive. Otherwise, the DMU is relatively unproductive. As addressed by Dittenhofer (2001), there are two reasons for measuring suppliers' productivity. One is because productivity is used to check whether or not a producer is performing satisfactorily. The second reason is that measuring productivity is a motivator for the producer. Productivity measurement may increase competition among producers (Azadi et al., 2015). Now, the fuzzy numbers are incorporated into Equation (4). Equation (4) can be developed to Equation (5) as follows:

$$
\begin{aligned}
&\max P = \left[\sum_{t=1}^{T} \eta_t \tilde{\mathrm{g}}_{to}(\alpha - \beta)\right] + \sum_{r=1}^{S} u_r \tilde{y}_{ro} && (5)\\
&\sum_{t=1}^{T} \eta_t \tilde{\mathrm{g}}_{to} = 1\\
&\sum_{r=1}^{S} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \le 0. && j = 1 \ldots m\\
&v_i \tilde{x}_{io} - \mu_i \le \frac{1}{m}. && i = 1 \ldots . m\\
&\frac{\propto}{S} - u_r \tilde{y}_{ro} + f_r \le 0. && \mathrm{R} = 1 \ldots . \mathrm{s}\\
&\sum_{i=1}^{m} \mu_i - \sum_{r=1}^{s} f_r - \beta \le 0\\
&\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{t=1}^{T} \eta_t \tilde{\mathrm{g}}_{tj} \le 0. && j = 1 \ldots n\\
&\propto . \beta . \mu_i . f_r . u_{r.} v_i . \eta_t \ge 0 && \forall \mathrm{i.r.t.}
\end{aligned}
$$

Where $\tilde{x}_{ij}(i = 1 \ldots . m)$,$\tilde{y}_{rj}(r = 1 \ldots . s)$ and $\tilde{\mathrm{g}}_{tj}(j = 1 \ldots . T)$ are fuzzy input, output, and goals of DMUs.This fuzzy integrated DEA model cannot be solved as a crisp model. To solve this model, many methods have been proposed, one of which is $\alpha -$ cut technique (Puri and Yadav, 2014;

Azadi et al., 2015; Wen and Li, 2009; Lozano 2014a, 2014b). α – Cut technique is a method that inputs and outputs are shown by different α – cut and different confidence intervals and levels. Each fuzzy coefficient can be viewed as a fuzzy variable, and each constraint can be considered a fuzzy event. Given the proposed model and the concept of the possibility space of the fuzzy event, some constraints are known as crisp values, and *others* are defined as uncertain values. For this reason, the objective function of the fuzzy integrated model can be written as follows:

$$\max \bar{\mathcal{G}} + \bar{\mathcal{F}} \tag{6}$$

$$\text{s.t:}$$

$$\pi\left(\sum_{t=1}^{T}\eta_t\tilde{g}_{to}(\alpha-\beta) \geq \bar{\mathcal{G}}\right) \geq \varepsilon_1$$

$$\pi\left(\sum_{r=1}^{s}u_r\tilde{y}_{ro} \geq \bar{\mathcal{F}}\right) \geq \varepsilon_2$$

Where ε_1 and ε_2 are predetermined acceptable levels of possibility for the two sections of the objective function. Therefore, the objective value $\bar{\mathcal{G}}$ is the maximum value that the return function $\sum_{t=1}^{T}\eta_t\tilde{g}_{to}(\alpha-\beta)$can be achieved at possibility level ε_1 or higher. Moreover, the objective value $\bar{\mathcal{F}}$ is the maximum value that the return function $\sum_{r=1}^{s}u_r\tilde{y}_{ro}$ can reach at the possibility level ε_2 or higher and is subject to the possibility levels of other fuzzy and crisp constraints. By adding the other constraints, the fuzzy integrated model can be reformulated by the following expression:

$$\max \mathrm{P} = \bar{\mathcal{G}} + \bar{\mathcal{F}} \tag{7}$$

$$\text{s.t:}$$

$$\pi\left(\sum_{t=1}^{T}\eta_t\tilde{g}_{to}(\alpha-\beta) \geq \bar{\mathcal{G}}\right) \geq \varepsilon_1$$

$$\pi\left(\sum_{r=1}^{s}u_r\tilde{y}_{ro} \geq \bar{\mathcal{F}}\right) \geq \varepsilon_2$$

$$\pi\left(\sum_{t=1}^{T}\eta_t\tilde{g}_{to} = 1\right) \geq \tau_1$$

$$\pi\left(\sum_{r=1}^{S}u_r\tilde{y}_{rj} - \sum_{i=1}^{m}v_i\tilde{x}_{ij} \leq 0\right) \geq \tau_2 \qquad j = 1 \dots m$$

$$\pi\left(v_i\tilde{x}_{io} - \mu_i \leq \frac{1}{m}\right) \geq \tau_3 \qquad i = 1 \dots . m$$

$$\pi\left(\frac{\propto}{s} - u_r\tilde{y}_{ro} + f_r \leq 0\right) \geq \tau_4 \qquad \mathrm{R} = 1 \dots . \mathrm{s}$$

$$\sum_{i=1}^{m}\mu_i - \sum_{r=1}^{s}f_r - \beta \leq 0$$

$$\pi\left(\sum_{r=1}^{s}u_r\tilde{y}_{rj} - \sum_{t=1}^{T}\eta_t\tilde{g}_{tj} \leq 0\right) \geq \tau_5 \qquad j = 1 \dots n$$

$$\propto . \beta . \mu_i . f_r . u_r . v_i . \eta_t \geq 0 \qquad \forall \mathrm{i.r.t.}$$

The related constraints should achieve the possible level where $\tau_1 \dots \tau_5$ are the predefined levels. In the crisp condition, the DMU will be relatively productive if the optimal value of Equation (4) equals 2. Meanwhile, the objective value $\left[\sum_{t=1}^{T}\eta_t\tilde{g}_{to}(\alpha-\beta)\right] + \sum_{r=1}^{S}u_r\tilde{y}_{ro}$ is the productive criterion of the DMU. Also $\bar{\mathcal{G}}$ and $\bar{\mathcal{F}}$ in the fuzzy integrated model are used to determine if the DMU is relatively productive at the predetermined possibility level. A DMU is productive if its $\mathrm{P} = \bar{\mathcal{G}} + \bar{\mathcal{F}}$ value is greater than or equals 2, otherwise, it is nonproductive. This Equation can be rewritten as follows:

$$\max \mathrm{P} = \bar{\mathcal{G}} + \bar{\mathcal{F}} \tag{8}$$

$$\text{s.t:}$$

$$(\alpha-\beta)\left(\sum_{t=1}^{T}\eta_t\tilde{g}_{to}\right)_{\varepsilon_1}^{u} \geq \bar{\mathcal{G}}$$

$$\left(\sum_{r=1}^{s}u_r\tilde{y}_{ro}\right)_{\varepsilon_2}^{u} \geq \bar{\mathcal{F}}$$

$$\left(\sum_{t=1}^{T}\eta_t\tilde{g}_{to}\right)_{\tau_1}^{u} \geq 1$$

$$\left(\sum_{t=1}^{T}\eta_t\tilde{g}_{to}\right)_{\tau_1}^{u} \leq 1$$

$$\left(\sum_{r=1}^{S}u_r\tilde{y}_{rj} - \sum_{i=1}^{m}v_i\tilde{x}_{ij}\right)_{\tau_2}^{L} \leq 0 \qquad j = 1 \dots m$$

$$(v_i\tilde{x}_{io} - \mu_i)_{\tau_3}^{L} \leq \frac{1}{m} \qquad i = 1 \dots . m$$

$$\left(\frac{\propto}{s} - u_r\tilde{y}_{ro} + f_r\right)_{\tau_4}^{L} \leq 0 \qquad \mathrm{R} = 1 \dots . \mathrm{s}$$

$$\sum_{i=1}^{m}\mu_i - \sum_{r=1}^{s}f_r - \beta \leq 0$$

$$\left(\sum_{r=1}^{s}u_r\tilde{y}_{rj} - \sum_{t=1}^{T}\eta_t\tilde{g}_{tj}\right)_{\tau_5}^{L} \leq 0 \qquad j = 1 \dots n$$

$$\propto . \beta . \mu_i . f_r . u_r . v_i . \eta_t \geq 0 \qquad \forall \mathrm{i.r.t.}$$

In the present study, we use triangular fuzzy numbers. Consider $\tilde{g}_{tj} = (g_{tj}^{a}.g_{tj}^{b}.g_{tj}^{c})$ is a fuzzy triangular number for the *t* goal of DMU_j; $\tilde{y}_{rj} = (y_{rj}^{a}.y_{rj}^{b}.y_{rj}^{c})$ is a triangular fuzzy number for the rth output ofDMU_j, and $\tilde{x}_{ij} = (x_{ij}^{a}.x_{ij}^{b}.x_{ij}^{c})$ is a triangular fuzzy number for the ith input of DMU_j. In this case, the linear programming model that transformed into the fuzzy model is presented as follows:

$$\max \mathrm{P} = \bar{\mathcal{G}} + \bar{\mathcal{F}} \tag{9}$$

$$s.t:$$

$$(\alpha-\beta)\sum_{t=1}^{T}\eta_t\left(g_{t1}^{c} - \varepsilon_1(g_{ti}^{c} - g_{ti}^{b})\right) \geq \bar{\mathcal{G}}$$

$$\sum_{r=1}^{S}u_r\left(y_{r1}^{c} - \varepsilon_2(y_{r1}^{c} - y_{r1}^{b})\right) \geq \bar{\mathcal{F}}$$

$$\sum_{t=1}^{T}\eta_t\left(g_{t1}^{c} - \tau_1(g_{t1}^{c} - g_{t1}^{b})\right) \geq 1$$

$$\sum_{t=1}^{T}\eta_t\left(g_{t1}^{c} - \tau_1(g_{t1}^{c} - g_{t1}^{b})\right) \leq 1$$

$$\sum_{r=1}^{S}u_r\left(y_{rj}^{a} + \tau_2(y_{rj}^{b} - y_{rj}^{a})\right) - \sum_{i=1}^{m}v_i\left(x_{ij}^{a} + \tau_2(x_{ij}^{b} - x_{ij}^{a})\right) \leq 0$$

$$v_i\left(x_{i1}^{a} + \tau_3(x_{i1}^{b} - x_{i1}^{a})\right) - \mu_i \leq \frac{1}{m}$$

$$\frac{\propto}{s} - u_r\left(y_{r1}^{a} + \tau_4(y_{r1}^{b} - y_{r1}^{a})\right) + f_r \leq 0$$

$$\sum_{r=1}^{S}u_r\left(y_{rj}^{a} + \tau_2(y_{rj}^{b} - y_{rj}^{a})\right) - \sum_{t=1}^{T}\eta_t\left(g_{tj}^{a} + \tau_5(g_{tj}^{b} - g_{tj}^{a})\right) \leq 0$$

$$\sum_{i=1}^{m}\mu_i - \sum_{r=1}^{s}f_r - \beta \leq 0$$

3. Result

This study used the model developed by Azadi et al. (2015) for ranking and measuring the efficiency of agricultural units in the region of Kabodarahang.

The required data was collected through a questionnaire in the villages of Kabodarahang in 2022–2021. To evaluate the validity and reliability of the questionnaire, Cronbach's alpha test was used after reviewing and confirming it with several subject matter experts. Its coefficient was estimated to be 0.81, which indicates the appropriate validity of the research tool. Then the data was analyzed using GAMS software.

The GAMS software has run this model. In this study, all the fuzzy constraints should be satisfied at the same possibility level. The results for five different possibility levels (0, 0.25, 0.5, 0.75, and 1) are presented in Table 2.

The results in Table 2 demonstrate that the three units of 3, 6, and 21 have the highest productivity at five probability levels. At the alpha level = 1, they have a productivity of 2 and are pretty efficient. Unit 22 has the lowest productivity rates of 0.67, 0.71, 0.75, 0.74, and 0.71 at five probability levels. By increasing the level of probabilities studied, the productivity of production units has also increased.

Table 2. Results of effectiveness, efficiency, and productivity indicators at different alpha levels

	Alpha=0			**Alpha=0.25**			**Alpha=0.5**		
No. DUMs	**Effectiveness**	**Efficiency**	**Productivity**	**Effectiveness**	**Efficiency**	**Productivity**	**Effectiveness**	**Efficiency**	**Productivity**
1	0.476	0.714	1.19	0.528	0.714	1.242	0.584	0.714	1.299
2	0.667	1	1.67	0.739	1	1.739	0.818	1	1.818
3	0.667	1	1.67	0.739	1	1.739	0.818	1	1.818
4	0.667	1	1.67	0.739	1	1.739	0.818	0.96	1.78
5	0.667	1	1.67	0.739	1	1.739	0.818	0.99	1.81
6	0.667	1	1.67	0.739	1	1.739	0.818	1	1.82
7	0.524	0.786	1.31	0.581	0.786	1.366	0.64	0.79	1.429
8	0.571	0.857	1.43	0.634	0.857	1.491	0.70	0.857	1.558
9	0.667	1	1.67	0.739	1	1.739	0.818	1	1.181
10	0.476	0.714	1.19	0.528	0.647	1.175	0.511	0.641	1.153
11	0.476	0.711	1.187	0.528	0.638	1.166	0.502	0.641	1.144
12	0.476	0.714	1.19	0.525	0.701	1.226	0.579	0.786	1.221
13	0.524	0.786	1.31	0.581	0.786	1.366	0.643	0.786	1.429
14	0.571	0.857	1.43	0.612	0.857	1.469	0.674	0.801	1.475
15	0.433	0.714	1.148	0.413	0.714	1.127	0.584	0.520	1.104
16	0.667	1	1.67	0.739	1	1.739	0.818	1	1.818
17	0.571	0.857	1.43	0.634	0.857	1.491	0.701	0.857	1.558
18	0.524	0.786	1.31	0.581	0.786	1.366	0.643	0.786	1.429
19	0.429	0.643	1.07	0.475	0.643	1.118	0.526	0.606	1.132
20	0.333	0.5	0.83	0.370	0.488	0.858	.409	0.409	0.818
21	0.667	1	1.67	0.739	1	1.739	0.818	1	1.818
22	0.286	0.429	0.714	0.317	0.429	0.745	0.336	0.411	0.746
23	0.571	0.857	1.43	0.634	0.824	1.458	0.589	0.857	1.446
24	0.476	0.714	1.19	0.528	0.654	1.182	0.446	0.714	1.160
25	0.550	0.857	1.41	0.634	0.754	1.387	0.511	0.857	1.368
26	0.554	0.857	1.401	0.634	0.757	1.390	0.522	0.857	1.379
27	0.407	0.714	1.12	0.528	0.572	1.10	0.363	0.714	1.077
28	0.381	0.571	0.95	0.422	0.571	0.994	0.468	0.571	1.039
29	0.571	0.857	1.43	0.634	0.857	1.491	0.639	0.857	1.496
30	0.381	0.571	0.95	0.422	0.513	0.935	0.330	0.571	0.901
31	0.667	0.955	1.62	0.79	0.882	1.622	0.697	0.924	1.622
32	0.381	0.558	0.94	0.422	0.485	0.907	0.378	0.496	0.874
33	0.571	0.827	1.40	0.531	0.857	1.388	0.520	0.857	1.377
34	0.476	0.714	1.19	0.480	0.714	1.194	0.457	0.714	1.171
35	0.381	0.558	0.94	0.336	0.571	0.907	0.302	0.571	0.874
36	0.604	1	1.60	0.604	1	1.604	0.604	1	1.604
37	0.619	0.929	1.55	0.686	0.929	1.615	0.760	0.929	1.688
38	0.571	0.857	1.43	0.634	0.857	1.491	0.701	0.857	1.558
39	0.619	0.929	1.55	0.686	0.929	1.615	0.760	0.929	1.668
40	0.571	0.929	1.55	0.686	0.929	1.615	0.760	0.929	1.688
41	0.619	0.857	1.43	0.634	0.857	1.491	0.701	0.857	1.558
42	0.524	0.786	1.31	0.581	0.786	1.366	0.643	0.786	1.429
43	0.524	0.786	1.31	0.581	0.786	1.366	0.643	0.786	1.429
44	0.476	0.714	1.19	0.528	0.672	1.20	0.584	0.593	1.178
45	0.571	0.857	1.43	0.634	0.851	10484	0.701	0.772	1.1473
46	0.381	0.571	0.95	0.422	0.571	0.994	0.468	0.571	1.039
47	0.476	0.714	1.19	0.528	0.714	1.242	0.584	0.714	1.299
48	0.571	0.857	1.43	0.634	0.587	1.491	0.701	0.857	1.558

Table 2 continued.

	Alpha=0.75			Alpha=1		
No. DUMs	**Effectiveness**	**Efficiency**	**Productivity**	**Effectiveness**	**Efficiency**	**Productivity**
1	0.646	0.714	1.409	0.714	0.694	1.361
2	0.889	1	1.889	1	0.889	1.889
3	0.905	1	2	1	1	1.905
4	0.771	1	1.766	1	0.766	1.771
5	0.818	1	1.822	1	0.822	1.818
6	0.905	1	2	1	1	1.905
7	0.711	0.786	1.57	0.786	0.786	1.497
8	0.776	0.857	1.70	0.857	0.845	1.633
9	0.853	1	1.86	1	0.860	1.853
10	0.414	0.714	1.10	0.714	0.387	1.128
11	0.405	0.714	1.09	0.714	0.377	1.119
12	0.496	0.714	1.19	0.714	0.473	1.210
13	0.711	0.786	1.57	0.786	0.786	1.497
14	0.743	0.732	1.47	0.821	0.650	1.475
15	0.614	0.466	1.05	0.672	0.381	1.080
16	0.895	1	1.90	0.957	0.947	1.895
17	0.776	0.857	1.71	0.857	0.857	1.633
18	0.711	0.786	1.57	0.786	0.786	1.497
19	0.582	0.531	1.08	0.643	0.439	1.113
20	0.452	0.322	0.727	0.5	0.227	0.775
21	0.905	1	2	1	1	1.905
22	0.336	0.371	0.67	0.336	0.336	0.707
23	0.577	0.857	1.42	0.563	0.857	1.434
24	0.421	0.714	1.11	0.394	0.714	1.135
25	0.491	0.857	1.33	0.471	0.857	1.348
26	0.510	0.857	1.35	0.496	0.857	1.367
27	0.338	0.714	1.026	0.311	0.714	1.053
28	0.479	0.571	1.03	0.458	0.571	1.05
29	0.639	0.857	1.49	0.636	0.857	1.497
30	0.293	0.571	0.82	0.252	0.571	0.864
31	0.697	0.924	1.62	0.688	0.934	1.622
32	0.265	0.571	0.796	0.224	0.571	0.836
33	0.507	0.857	1.35	0.494	0.857	1.364
34	0.432	0.714	1.12	0.405	0.714	1.147
35	0.265	0.571	0.80	0.224	0.571	0.836
36	0.604	1	1.604	0.604	1	1.604
37	0.840	0.929	1.86	0.929	0.929	1.769
38	0.776	0.857	1.71	0.857	0.857	1.633
39	0.840	0.929	1.86	0.929	0.929	1.769
40	0.840	0.929	1.86	0.929	0.929	1.769
41	0.776	0.857	1.71	0.857	0.857	1.633
42	0.711	0.786	1.57	0.786	0.786	1.497
43	0.711	0.786	1.54	0.773	0.771	1.497
44	0.646	0.507	1.13	0.701	0.424	1.153
45	0.776	0.685	1.45	0.844	0.603	1.461
46	0.517	0.536	1.03	0.571	0.462	1.053
47	0.646	0.695	1.33	0.714	0.620	1.342
48	0.768	0.857	1.62	0.767	0.857	1.626

According to Table 3, the maximum effectiveness value is 0.82, which is related to units (2, 3, 4, 5, and 6). The maximum efficiency value is 1 (for units 2, 3, and 6); the maximum productivity value among the production units under study is 1.82 (for unit 6). The minimum of these indicators is equal to 0.32, 0.41, and 0.72, and their average is equal to 0.61, 0.78, and 1.37, respectively. According to the results, 14% of production units are efficient. Units 2 to 6, 16, and 21 are superior to other units in terms of the effectiveness index. Units 2, 3, 6, 9, 16, 21, and 36 are efficient units. In total, unit 6 has the highest productivity and is the most stable unit among the other units. Therefore, this unit was selected as the best producer. Furthermore, units 35, 22, 32, and 20 have the lowest efficiencies.

The effectiveness of production units increases with increasing probability levels; in other words, production units get closer to their goals. Likewise, the productivity index increases with increasing probability level and efficiency decreases. The Table 3 shows that potato fields have an average efficiency of 0.80, 0.78, 0.79, 0.78, and 0.73% at different probability levels. In other words, potato production can be increased by using the same amount of input; efficiency will also increase by 20, 22, 21, 22, and 27% at different probability levels. The difference between the lowest and highest levels of efficiency shows that there are many differences between farmers in the region, which can be reduced by various methods, including the introduction of sample farmers. Table 4 show the result of Frequency distribution of effectiveness, efficiency and productivity of potato fields in different alpha level.

Table 3. Summary assessment result of effectiveness, efficiency and productivity for farms in sample in different levels.

	Effectiveness				
	$\boldsymbol{\alpha = 0}$	$\boldsymbol{\alpha = 0.25}$	$\boldsymbol{\alpha = 0.5}$	$\boldsymbol{\alpha = 0.75}$	$\boldsymbol{\alpha = 1}$
Average	0.53	0.58	0.61	0.64	0.70
Max	0.67	0.79	0.82	0.91	1.00
Min	0.29	0.32	0.30	0.27	0.22
Standard deviation	0.10	0.12	0.15	0.19	0.23
	Efficiency				
	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
Average	0.80	0.78	0.79	0.78	0.73
Max	1.00	1.00	1.00	1.00	1.00
Min	0.43	0.43	0.41	0.32	0.23
Standard deviation	0.15	0.16	0.16	0.18	0.20
	Productivity				
	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
Average	1.33	1.37	1.37	1.43	1.42
Max	1.67	1.74	1.82	2.00	1.91
Min	0.71	0.75	0.75	0.67	0.71
Standard deviation	0.25	0.27	0.29	0.37	0.33

Table 4. Frequency distribution of effectiveness, efficiency and productivity of potato fields

	Effectiveness				
	$\boldsymbol{\alpha = 0}$	$\boldsymbol{\alpha = 0.25}$	$\boldsymbol{\alpha = 0.5}$	$\boldsymbol{\alpha = 0.75}$	$\boldsymbol{\alpha = 1}$
Less than 50%	18	10	10	12	11
Between 50% to 60%	8	14	21	5	3
Between 60% to 70%	12	15	9	7	5
More than 70%	0	9	8	23	29
	Efficiency				
	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
Less than 50%	8	14	12	12	17
Between 50% to 60%	15	12	12	13	9
Between 60% to 70%	11	10	11	10	13
More than 70%	13	12	13	13	9
	Productivity				
	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
Less than 1	7	7	5	5	5
Between 1 to 1.5	28	28	26	20	25
Between 1.5 to 2	13	13	17	23	18

4. Conclusion

Due to the importance of potato crops in Kabodarahang city, an attempt was made to measure potato fields' efficiency, effectiveness, and productivity. The presented fuzzy model in this paper is a method for dealing with fuzzy data. To solve the fuzzy model, the insoluble fuzzy model must be converted to a solvable linear model, for which the probability method is used. The findings indicate that the potato growers of Kabodarahang are in good condition in terms of efficiency. In addition, they can increase their efficiency by reducing the use of inputs without reducing the product to avoid wasting production inputs and be placed on the verge of production efficiency. The efficiency index shows that the efficiency values are different at different levels of α. As the probability level increases, the unit's effectiveness increases, the value of efficiency decreases, and the value of productivity increases. Increasing efficiency improves productivity and contributes to achieving the goals of the production unit. According to the results, 14% of the producers are efficient and the rest are on the verge of efficiency. In addition, 75% of the producers have an efficiency of between 70% and 100%. In total, unit 6 has the highest productivity and is the most stable unit among the other units. Thus, this unit was selected as the best producer. An examination of efficiency at different levels of alpha shows that producers are at a high level in terms of efficiency. Therefore, to increase production, production technology should be improved. Thus, politicians and policymakers should consider the new planting, growing, and harvesting technologies.

Reference

Alinezhad, A.R., Kakavand, H., Kazemi, A., Shakorloo, A. 2018. Presenting a hybrid model of data envelopment analysis and multi-objective decision making to achieve the efficiency score of decision making units in grey environment. Journal of Development Evolution Management, 31, 43-49. (In Persian).

Azadi, M., Jafarian, M., Farzinpooor Saen, k., Mirhedayatian, S.M., 2015. Anew fuzzy DEA model for evaluation of efficiency and effectiveness of suppliers in sustainable supply chain management context. Computer and Operation Research. 54, 274-285.

Bellman, R.E., Zadeh, L.A., 1970. Decision-making in a fuzzy environment. Management Science, 17(4), B-141.

Charnes, A., Cooper, W. W., Rhodes, E. 1978. Measuring the efficiency of decision making units. European Journal of Operational Research, 2(6), 429-444.

Chiou, Y.C., Lan, L.W., Yen, B.T., 2010. A joint measurement of efficiency and effectiveness for nonstorable commodities: Integrated data envelopment analysis approaches. European Journal of Operational Research, 201(2), 477-489.

Cooper, W.W., Park, K.S. and Yu, G., 1999. IDEA and AR-IDEA: models for dealing with imprecise data in DEA. Management Science, 45(4), 597-607.

Dadmand, F., Naji Azimi, Z., 2018. The application of fuzzy dea in evaluating the efficiency of wheat production case study: city of Torbate Hydarieh. Journal of Agricultural Economics Research, 10(1), 87-109. (In Persian).

Dittenhofer, M., 2001. Internal auditing effectiveness: an expansion of present methods. Managerial Auditing Journal, 16(8), 443-450.

Esmaeili, M., 2012. An enhanced Russell measure in DEA with interval data. Applied Mathematics and Computation, 219(4), 1589-1593.

Farzipoor Saen, R., 2009. A decision model for ranking suppliers in the presence of cardinal and ordinal data, weight restrictions, and nondiscretionary factors. Annals of Operations Research, 172(1), 177-192.

Guo, P., Tanaka, H., 2001. Fuzzy DEA: a perceptual evaluation method. Fuzzy Sets and Systems, 119(1), 149-160.

Hatami-Marbini, A., Ebrahimnejad, A., Lozano, S., 2017. Fuzzy efficiency measures in data envelopment analysis using lexicographic multi objective approach. Computers & Industrial Engineering, 105, 362-376.

Hatami-Marbini, A., Saati, S., Tavana, M., 2011. Data envelopment analysis with fuzzy parameters: an interactive approach. International Journal of Operations Research and Information Systems (IJORIS), 2(3), 39-53.

Hatami-Marbini, A., Tavana, M., Emrouznejad, A., Saati, S., 2012. Efficiency measurement in fuzzy additive data envelopment analysis. International Journal of Industrial and Systems Engineering, 10(1), 1-20.

Kao, C., Liu, S.T., 2003. A mathematical programming approach to fuzzy efficiency ranking. International Journal of Production Economics, 86(2), 145-154.

Kumar, A., Jain, V., Kumar, S., 2014. A comprehensive environment friendly approach for supplier selection. Omega, 42(1), 109-123.

Lozano, S., 2014a. Process efficiency of two-stage systems with fuzzy data. Fuzzy Sets and Systems, 234, 36-49.

Lozano, S., 2014b. Computing fuzzy process efficiency in parallel systems. Fuzzy Optimization and Decision Making, 13(1), 73-89.

Mardani, M., Mirzaei, A., Ohadi, N., 2020. Investigating the Rice Energy Efficiency Using Interval Fuzzy Data Envelopment Analysis Model (Case Study: Rice Farmers in Golestan Province). Iranian Journal of Agricultural Economic and Development, 51(4), 663-667. (In Persian).

Mardani, M., Ziaee, S., 2016. Determining the Efficiency of Irrigated Wheat Farms in Neyshabur County under Uncertainty. Journal of Agricultural Economics and Development, 3(2), 136-147. (In Persian).

Ministry of Jahad Agriculture, 2011. Agricultural Statistics of Crops, Deputy of Planning and Economy, Information and Communication Technology Center, (In Persian).

Nandy, A., Singh, P.K., 2021. Application of fuzzy DEA and machine learning algorithms in efficiency estimation of paddy producers of rural Eastern India. Benchmarking an International Journal, 28(1), 229-248.

Puri, J., Yadav, S.P., 2014. A fuzzy DEA model with undesirable fuzzy outputs and its application to the banking sector in India. Expert Systems with Applications, 41(14), 6419-6432.

Qin, R., Liu, Y.K., 2010. A new data envelopment analysis model with fuzzy random inputs and outputs. Journal of Applied Mathematics and Computing, 33(1), 327-356.

Sengupta, J.K., 1992. A fuzzy systems approach in data envelopment analysis. Computers and Mathematics with Applications, 24(8-9), 259-266.

Sepehrdoust, H., Emami, S.A.S., 2017. Comparative advantage of potato production and related government policies in Hamedan. Journal of Agricultural Economics Research, 33, 51-70. (In Persian).

Tlig, H., Hamed, A.B., 2017. Assessing the efficiency of commercial Tunisian banks using fuzzy data envelopment analysis. Journal of Data Envelopment Analysis and Decision Science, 2, 14-27.

Toma, E., Dobre, C., Dona, I., Cofas, E., 2015. DEA applicability in assessment of agriculture efficiency on areas with similar geographically patterns. Agriculture and Agricultural Science Procedia, 6, 704-711.

Weber, C.A., Current, J., Desai, A., 2000. An optimization approach to determining the number of vendors to employ. Supply Chain Management: An International Journal, 5(2), 90-98.

Wen, M., Li, H., 2009. Fuzzy data envelopment analysis (DEA): model and ranking method. Journal of Computational and Applied Mathematics, 223(2), 872-878.